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Diffraction by a circular aperture in a non-planar screen

H. S. TAN
Department of Physics, University of Malaya, Kuala Lumpur, Malaysia
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Abstract. An edge-current method is formulated for the problem of diffraction by a circular aperture formed by the envelope of a system of wedges. Some experimental results are presented for comparison. For various non-planar screens it is found that the aperture field is not changed appreciably by changes in the geometry of the screen on the image side, or by changes in the edge geometry. However, the geometry of the screen on the source side has a large influence on the aperture field.

1. Introduction

Braunbek (1959) has derived approximate formulae for the electric and magnetic fields **E** and **H** on the axis of a circular aperture formed by the envelope of a system of half-planes inclined at an angle α to the axis. This problem of diffraction by an aperture in a thin non-planar screen is important in that it enables a comparison to be made with an identical aperture in a plane screen, so that the influence of the screen geometry on the diffracted field can be deduced. Braunbek's method was to assume Kirchhoff boundary values for **E** and **H** in the aperture, and to add corrections by fitting half-plane values of **E** and **H** over a small annulus on the non-planar screen and in the aperture. The diffracted field everywhere can then be calculated using the equivalence theorem, but considerable labour is involved, except on the axis, where the calculations simplify somewhat.

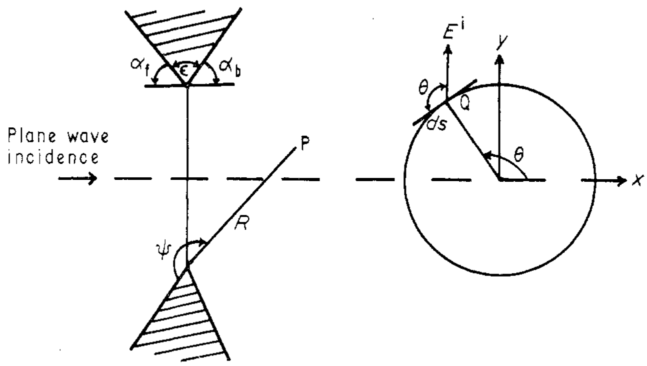

Figure 1. Circular aperture formed by a system of wedges.

It is the purpose of this paper to extend this problem to that of diffraction by a circular aperture formed by the envelope of a system of wedges, shown in figure 1. Braunbek's thin conal screen is the case when the internal wedge angle $\epsilon = 0$. We use an edge-current theory generalized from Millar's (1955) method. This is simpler than Braunbek's method, and for the thin conal screen gives identical results for the axial field. Moreover, the field in the aperture and off-axis can be calculated readily. By varying the angles of inclination α_f and α_b, and the wedge angle ϵ, comparison can be made for diffraction by various screens, so that the influence of both screen geometry and edge geometry can be seen. Some measurements on such apertures are presented for comparison with the edge-current theory, and with approximations using the first and second Rayleigh integrals.

2. Edge-current theory

Millar's method is based on the fact that in certain regions of the far field of a half-plane with E- (or H-) polarized plane-wave incidence, the E (or H) field is asymptotically a cylindrical wave diverging from the diffracting edge. This cylindrical wave can be considered to have been radiated by fictitious electric and magnetic currents on the half-plane edge. In the aperture diffraction problem, the aperture edge at every point is considered to be a tangential half-plane, with these electric and magnetic currents flowing along the rim. The combined radiated fields of these currents then give the diffracted field of the aperture. This approach can be extended simply to apertures whose edges at every point are considered to be wedges instead of half-planes.

For a plane wave with electric vector (0, 1, 0) $\exp(-ikz)$ normally incident on a wedge whose edge makes an angle θ with the y axis, the diffracted far field everywhere in space, except near the geometrical-optics shadow region and the reflected-ray region, is given by the asymptotic expressions (Jones 1964)

$$\left.\begin{matrix}\mathbf{E}^{\mathrm{d}}\\ \mathbf{H}^{\mathrm{d}}\end{matrix}\right\} = \left\{\begin{matrix}(-\sin\theta\cos\theta,\ \cos^2\theta,\ 0)D_{\mathrm{e}}(\alpha_{\mathrm{f}},\ \psi)\\ (-\sin^2\theta,\ \sin\theta\cos\theta,\ 0)D_{\mathrm{m}}(\alpha_{\mathrm{f}},\ \psi)\end{matrix}\right. \frac{\exp(-ikR-\frac{1}{4}i\pi)}{(\pi kR)^{1/2}} \tag{1}$$

where, referring to figure 1,

$$\left.\begin{matrix}D_{\mathrm{e}}(\alpha_{\mathrm{f}},\ \psi)\\ D_{\mathrm{m}}(\alpha_{\mathrm{f}},\ \psi)\end{matrix}\right\} = \frac{1}{\surd 2}\frac{\pi}{2\pi-\epsilon}\left[\pm\frac{\sin\{\pi^2/(2\pi-\epsilon)\}}{\cos\{\pi^2/(2\pi-\epsilon)\}-\cos\{\pi/(2\pi-\epsilon)\}(\psi-\alpha_{\mathrm{f}})}\right.$$
$$\left.-\frac{\sin\{\pi^2/(2\pi-\epsilon)\}}{\cos\{\pi^2/(2\pi-\epsilon)\}-\cos\{\pi/(2\pi-\epsilon)\}(\psi+\alpha_{\mathrm{f}})}\right]. \tag{2}$$

With such a plane wave normally incident on the aperture in figures 1 and 2, the edge currents on a line element ds at a point Q on the aperture rim are

$$\left.\begin{matrix}d\mathbf{I}_{\mathrm{e}}\\ d\mathbf{I}_{\mathrm{m}}\end{matrix}\right\} = \left\{\begin{matrix}D_{\mathrm{e}}\cos\theta\\ D_{\mathrm{m}}\sin\theta\end{matrix}\right. (-\mathbf{n}_x\sin\theta+\mathbf{n}_y\cos\theta)\,ds. \tag{3}$$

No restrictions have been made on the shape of the aperture, except that the radius of curvature at every point be large enough for the rim locally to approximate that of a wedge.

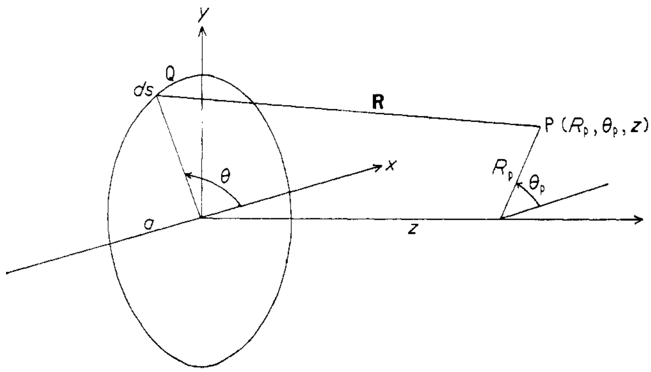

Figure 2. Diffracted field at P.

The diffracted field of the aperture is thus given by a line integral along the rim of the fields radiated by $d\mathbf{I}_{\mathrm{e}}$ and $d\mathbf{I}_{\mathrm{m}}$ in equation (3). At a point P (R_{p}, θ_{p}, z) the radiated electric field due to $\mathbf{I}_{\mathrm{e}}$ and $\mathbf{I}_{\mathrm{m}}$ is

$$\mathbf{E}(\mathrm{P}) = \mathbf{n}_y\,\mathrm{e}^{-ikz}+\frac{1}{\surd 2\pi}\oint_{\mathrm{rim}}\frac{\exp(-ikR)}{R}(D_{\mathrm{m}}\sin\theta+D_{\mathrm{e}}\cos\theta\,\mathbf{n}_R\times)\{\mathbf{n}_R\times(-\mathbf{n}_x\sin\theta$$
$$+\mathbf{n}_y\cos\theta)\}\,ds \tag{4}$$

where $\mathbf{n}_R$ is a unit vector along $\mathbf{R}$, and $\mathbf{n}_x$, $\mathbf{n}_y$ and $\mathbf{n}_z$ are unit vectors along the coordinate axes. Equation (4) applies to an aperture of general shape; for a circular aperture of radius a, the expression for the main polarization component of the electric field E_y becomes

$$E_y(\mathrm{P}) = \exp(-ikz) - \frac{a}{\sqrt{2\pi}} \int_0^{2\pi} \frac{\exp(-ikR)}{R^2} \left[z \left(D_\mathrm{m} \sin^2\theta + D_\mathrm{e} \frac{z}{R} \cos^2\theta \right) \right.$$
$$\left. + D_\mathrm{e} \frac{R_\mathrm{p} \cos\theta_\mathrm{p} \cos\theta - a\cos^2\theta}{R} \{R_\mathrm{p} \cos(\theta - \theta_\mathrm{p}) - a\} \right] d\theta. \tag{5}$$

After some manipulation we obtain E_y in a form suitable for numerical computation:

$$E_y(\mathrm{P}) = \left[\exp(-ikz) - \frac{a}{\sqrt{2\pi}} \left\{ \int_0^{\pi} \frac{\exp(-ikR')}{R'} D_\mathrm{e}\, d\theta - R_\mathrm{p}^2 \int_0^{\pi} \frac{\exp(-ikR')}{(R')^3} D_\mathrm{e} \sin^2\theta\, d\theta \right. \right.$$
$$\left. + z \int_0^{\pi} \frac{\exp(-ikR')}{(R')^2} D_\mathrm{m}\, d\theta \right\} - \frac{a}{\sqrt{2\pi}} \cos 2\theta_\mathrm{p} \left\{ \int_0^{\pi} \frac{\exp(-ikR')}{R'} D_\mathrm{e} \cos 2\theta\, d\theta \right.$$
$$+ R_\mathrm{p}^2 \int_0^{\pi} \frac{\exp(-ikR')}{(R')^3} D_\mathrm{m} \sin^2\theta\, d\theta$$
$$\left. \left. - 2aR_\mathrm{p} \int_0^{\pi} \frac{\exp(-ikR')}{(R')^3} D_\mathrm{e} \cos\theta \sin^2\theta\, d\theta - z \int_0^{\pi} \frac{\exp(-ikR')}{(R')^2} D_\mathrm{m} \cos 2\theta\, d\theta \right\} \right] \tag{6}$$
$$R' = (z^2 + R_\mathrm{p}^2 + a^2 - 2aR_\mathrm{p} \cos\theta)^{1/2}.$$

The restriction on equations (5) and (6) is that the point P must not lie in or near those regions where the asymptotic expressions in equations (1) and (2) are invalid. For a half-plane, these regions are two parabolae along the reflected ray and the transmitted ray (Born and Wolf 1965). Another restriction is that geometrical-optics reflected rays on the screen must not pass through the aperture (Braunbek 1959), i.e. $\alpha_\mathrm{f} > 45°$.

The other polarization components E_x and E_z are generally much smaller than E_y. Attempts to measure them were unsuccessful, since they are of the order of 20 dB smaller than E_y.

3. Axial and aperture fields

For points on the axis the terms in equation (5) simplify to give

$$E_y(0, 0, z) = \exp(-ikz) - \frac{a \exp\{-ik(a^2+z^2)^{1/2}\}}{\{2(a^2+z^2)\}^{1/2}} \left\{ -D_\mathrm{e} + D_\mathrm{m} \frac{z}{(a^2+z^2)^{1/2}} \right\} \tag{7}$$

which is identical with Braunbek's result for a thin conal screen ($\epsilon = 0$). For the thin plane screen equation (7) agrees with the results of Millar (1955) and Franz (1957). The edge-current theory can be improved by considering interaction between the fields diffracted by diametrically opposite points on the aperture rim, in an analogous way to Millar's (1956) method.

For the aperture field we have

$$E_y(R_\mathrm{p}, \theta_\mathrm{p}, 0) = 1 + \frac{aD_\mathrm{e}}{\sqrt{2\pi}} \int_0^{2\pi} \frac{\exp(-ik\rho)}{\rho} \cos(\gamma - \theta) \cos\theta \cos\gamma\, d\theta$$
$$\rho = \{R_\mathrm{p}^2 + a^2 - 2aR_\mathrm{p} \cos(\theta - \theta_\mathrm{p})\}^{1/2} \tag{8}$$
$$\gamma = \theta_\mathrm{p} + \pi - \sin^{-1}\left\{ \frac{a}{\rho} \sin(\theta - \theta_\mathrm{p}) \right\}.$$

It is seen that the aperture field depends on the angles α_f, α_b and ϵ in a simple manner, the factor D_e being outside the integral. D_m does not appear in equation (8) because, to this order of approximation, the magnetic currents on the edge do not contribute to the aperture field.

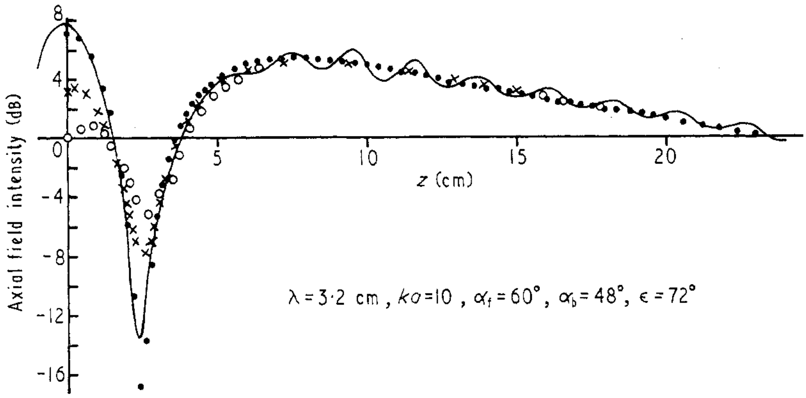

Figure 3. Axial field E_y of circular aperture: full circles, edge-current theory, equation (7); crosses, second Rayleigh integral, equation (10); open circles, first Rayleigh integral, equation (9); full curve, experiment.

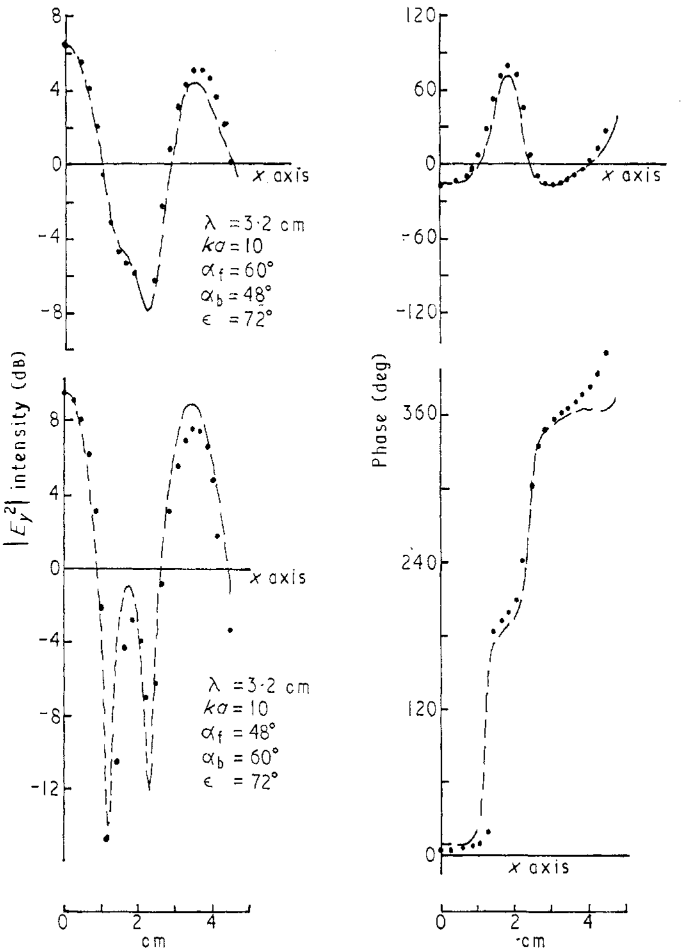

Figure 4. H-plane aperture electric field E_y of wedge-shaped screen: full circles, edge-current theory, equation (8); broken curves, experiment.

Finally, we quote the formulae for the axial field given by the first and second Rayleigh integrals, in which it is assumed that the electric and magnetic fields in the aperture are the incident fields $\mathbf{E}^i$ and $\mathbf{H}^i$, respectively (Bouwkamp 1954):

$$E_{1y}(0,\,0,\,z) = \exp(-ikz) - \frac{z}{(a^2+z^2)^{1/2}} \exp\{-ik(a^2+z^2)^{1/2}\} \tag{9}$$

$$E_{2y}(0,\,0,\,z) = \exp(-ikz) - \frac{1}{2}\left\{1 + \frac{z^2}{a^2+z^2} + \frac{ia^2}{k(a^2+z^2)^{3/2}}\right\} \exp\{-ik(a^2+z^2)^{1/2}\}. \tag{10}$$

These are well-known approximations in planar diffraction theory, and E_{2y} has been shown to give good results for plane apertures.

4. Results and conclusions

Measurements of aperture electric field intensity and phase have been made on aluminium screens inside a microwave anechoic room, using a small slot-fed electric dipole as probe at 3·2 cm wavelengths. The results are shown in figures 3, 4 and 5 for comparison

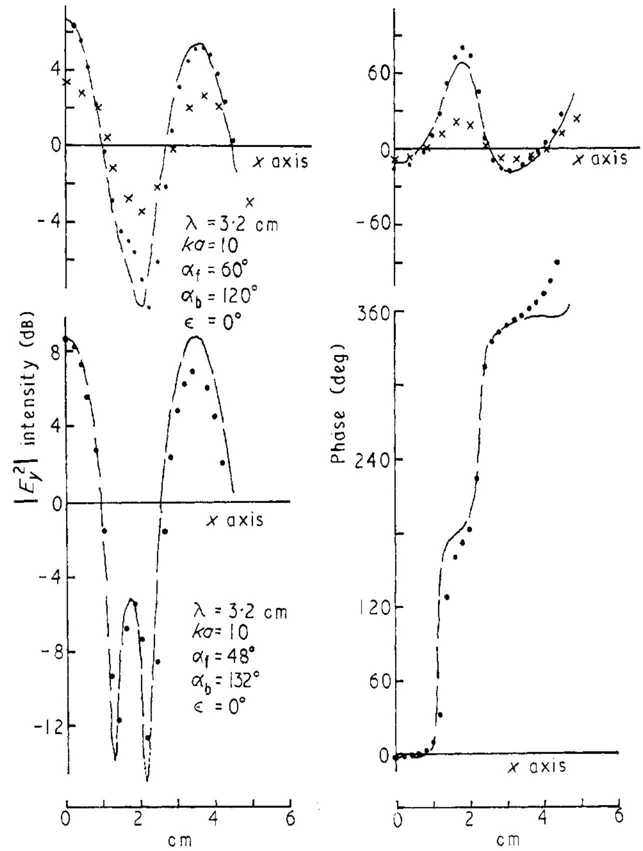

Figure 5. H-plane aperture electric field E_y of thin, conal screen: full circles, edge-current theory, equation (8); crosses, second Rayleigh integral; broken curves, experiment.

with theory. In general, the edge-current theory gives good agreement, except for the small ripples in the axial field, which are thought to be a reflection interaction effect similar to that found in a non-planar slit aperture problem (Tan 1967 a, b). Approximate theories based on concepts of edge diffraction, like those of Millar, Braunbek, Keller (1962) and Kirchhoff, are unable to predict this reflection effect, which is a screen interaction.

The graphs also indicate that α_f has a much larger effect on the aperture field than α_b and ϵ. For a given α_f, there is practically no difference between a thin screen with $\epsilon = 0$ and a wedge-shaped screen with $\epsilon = 72°$. But changing α_f from 60° to 48° makes a drastic difference. The reason for this is that the factors D_e and D_m are more sensitive to changes in α_f than in α_b and ϵ. It appears therefore that both the screen geometry on the image side, as well as the edge geometry, do not influence the aperture fields much. Finally, both Rayleigh integrals give poor results in the aperture, but are reasonably good (except for the reflection interaction) for axial fields about two wavelengths behind the aperture.

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